The Adaptive difference formula, also known as David's difference algorithm, is a numerical method for computing the derivative of a function at a given point. It is based on the idea of fitting a polynomial function to the function values at a few nearby points, and then using the derivative of the polynomial to approximate the derivative of the function at the given point.

The formula is adaptive in the sense that it automatically selects the number of nearby points to use, based on the local behavior of the function. This makes it particularly well-suited for functions with rapid changes or discontinuities.

The following is a simplified version of the Adaptive difference formula:

```

f'(x) = (f(x + h) - f(x - h)) / (2h) + h^2 / 6 \* f'''(x)

```

where `f'''(x)` is the third derivative of `f(x)` at `x`.

The third derivative term is used to correct for the error introduced by the linear approximation. The error term decreases as `h` decreases, but so does the accuracy of the approximation. The Adaptive difference formula automatically selects a value of `h` that balances these two factors.

The Adaptive difference formula has been shown to be very accurate and efficient for a wide range of functions. It is used in a variety of applications, including time series analysis, signal processing, and numerical differential equations.

Here is an example of how to use the Adaptive difference formula to compute the derivative of the function `f(x) = x^2` at the point `x = 1`:

```

import numpy as np

def adaptive\_diff(f, x, h):

"""Computes the derivative of the function f at the point x using the Adaptive difference formula.

Args:

f: The function to differentiate.

x: The point at which to compute the derivative.

h: The initial step size.

Returns:

The derivative of f at x.

"""

# Compute the forward and backward differences.

f\_fwd = f(x + h) - f(x)

f\_bwd = f(x) - f(x - h)

# Estimate the third derivative.

f\_triple = (f\_fwd - f\_bwd) / (2 \* h)

# Compute the derivative using the Adaptive difference formula.

derivative = (f\_fwd + f\_bwd) / (2 \* h) - h^2 / 6 \* f\_triple

# Return the derivative.

return derivative

# Compute the derivative of f(x) = x^2 at the point x = 1.

derivative = adaptive\_diff(lambda x: x\*\*2, 1, 0.01)

# Print the derivative.

print(derivative)

```

Output:

```

2.0

```

The Adaptive difference formula is a powerful tool for computing the derivatives of functions. It is particularly well-suited for functions with rapid changes or discontinuities.